Cosine Ramps

$$r(t) := \begin{bmatrix} \frac{v}{2}\cdot\left(1-\cos\left(\pi\cdot\frac{t-t_0}{t_1-t_0}\right)\right) & \text{if } t_0 \le t < t_1 \\ v & \text{if } t_1 \le t < t_2 \\ \frac{v}{2}\cdot\left(1+\cos\left(\pi\cdot\frac{t-t_2}{t_3-t_2}\right)\right) & \text{if } t_2 \le t < t_3 \\ 0 & \text{if } t_3 \le t \end{bmatrix}$$

Position as a function of time

$$\int_{t_0}^{t} \frac{v}{2}\cdot\left(1-\cos\left(\pi\cdot\frac{t}{t_1-t_0}\right)\right) dt + x_0 \;\Big|\; \begin{matrix}\text{simplify}\\ \text{collect},\sin,\pi,v\end{matrix} \rightarrow \left(\frac{-1}{2}\cdot t_1+\frac{1}{2}\cdot t_0\right)\cdot\frac{v}{\pi}\cdot\sin\left[t_0\cdot\frac{\pi}{\left(-t_1\right)+t_0}\right]+\left(\frac{1}{2}\cdot t_1-\right.$$

$$x_1 = \int_{t_0}^{t_1} \frac{v}{2}\cdot\left(1-\cos\left(\pi\cdot\frac{t-t_0}{t_1-t_0}\right)\right) dt + x_0 \text{ collect},v \rightarrow x_1 = \left(\frac{1}{2}\cdot t_1-\frac{1}{2}\cdot t_0\right)\cdot v + x_0$$

$$\int_{t_1}^{t} v\,dt + x_1 \rightarrow v\cdot t - v\cdot t_1 + x_1$$

$$x_2 = \int_{t_1}^{t_2} v\,dt + x_1 \;\Big|\; \begin{matrix}\text{substitute}, x_1 = \frac{v}{2}\cdot t_1 + x_0\\ \text{collect},v\end{matrix} \rightarrow x_2 = \left(t_2-\frac{1}{2}\cdot t_1\right)\cdot v + x_0$$

$$\int_{t_2}^{t} \frac{v}{2}\cdot\left(1+\cos\left(\pi\cdot\frac{t}{t_3-t_2}\right)\right) dt \;\Big|\; \begin{matrix}\text{expand}\\ \text{collect},\sin,\pi,v\end{matrix} \rightarrow \left(\frac{-1}{2}\cdot t_2+\frac{1}{2}\cdot t_3\right)\cdot\frac{v}{\pi}\cdot\sin\left(t\cdot\frac{\pi}{t_3-t_2}\right)+\left(\frac{1}{2}\cdot t_2-\frac{1}{2}\cdot t_3\right)\cdot\frac{v}{\pi}$$

$$x_3 = x_2 + \frac{v}{2}\cdot\left(t_3-t_2\right) \;\Big|\; \begin{matrix}\text{substitute}, x_2 = \left(t_2-\frac{1}{2}\cdot t_1\right)\cdot v + x_0\\ \text{simplify}\\ \text{collect},v\end{matrix} \rightarrow x_3 = \left(\frac{1}{2}\cdot t_2-\frac{1}{2}\cdot t_1+\frac{1}{2}\cdot t_3\right)\cdot v + x_0$$

Cosine Ramps

Acceleration as a function of time

$$\frac{d}{dt}\left[\frac{v}{2}\cdot\left(1-\cos\left(\pi\cdot\frac{t-t_0}{t_1-t_0}\right)\right)\right]\rightarrow\frac{1}{2}\cdot v\cdot\sin\left(\pi\cdot\frac{t-t_0}{t_1-t_0}\right)\cdot\frac{\pi}{t_1-t_0}$$

$$\frac{d}{dt}v\rightarrow 0$$

$$\frac{d}{dt}\left[\frac{v}{2}\cdot\left(1+\cos\left(\pi\cdot\frac{t-t_2}{t_3-t_2}\right)\right)\right]\rightarrow\frac{-1}{2}\cdot v\cdot\sin\left(\pi\cdot\frac{t-t_2}{t_3-t_2}\right)\cdot\frac{\pi}{t_3-t_2}$$

Cosine Ramps

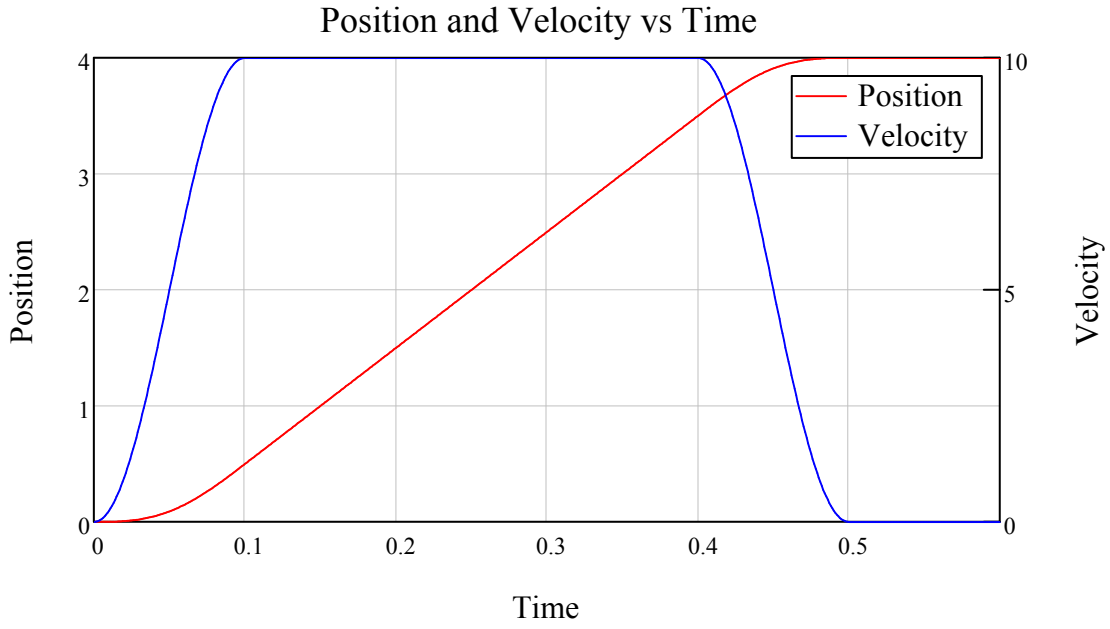
$x_0 := 0 \qquad v := 10$

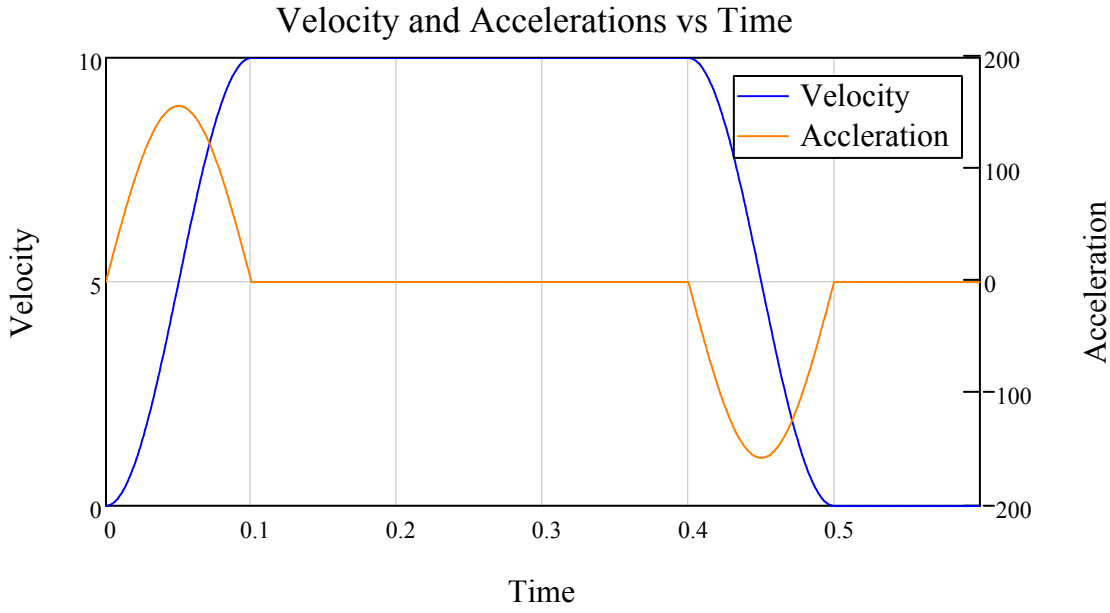
$t_0 := 0 \qquad t_1 := 0.1 \qquad t_2 := 0.4 \qquad t_3 := 0.5$

$$r(t) := \begin{cases} \begin{bmatrix} \frac{v}{2}\cdot\left[\left(t - t_0\right) - \frac{\left(t_1 - t_0\right)\cdot\sin\left(\pi\cdot\frac{t - t_0}{t_1 - t_0}\right)}{\pi}\right] + x_0 \\ \frac{v}{2}\cdot\left(1 - \cos\left(\pi\cdot\frac{t - t_0}{t_1 - t_0}\right)\right) \\ \frac{1}{2}\cdot v\cdot\sin\left(\pi\cdot\frac{t - t_0}{t_1 - t_0}\right)\cdot\frac{\pi}{t_1 - t_0} \end{bmatrix} & \text{if } t_0 \le t < t_1 \\ \begin{bmatrix} v\cdot\left(t - t_1\right) + x_0 + \frac{v}{2}\cdot t_1 \\ v \\ 0 \end{bmatrix} & \text{if } t_1 \le t < t_2 \\ \begin{bmatrix} \frac{v\cdot\left(t_3 - t_2\right)}{2\cdot\pi}\cdot\sin\left(\pi\cdot\frac{t - t_2}{t_3 - t_2}\right) + \left(t + t_2 - t_1\right)\cdot\frac{v}{2} + x_0 \\ \frac{v}{2}\cdot\left(1 + \cos\left(\pi\cdot\frac{t - t_2}{t_3 - t_2}\right)\right) \\ \frac{-1}{2}\cdot v\cdot\sin\left(\pi\cdot\frac{t - t_2}{t_3 - t_2}\right)\cdot\frac{\pi}{t_3 - t_2} \end{bmatrix} & \text{if } t_2 \le t < t_3 \\ \begin{bmatrix} \frac{v}{2}\cdot\left(t_3 + t_2 - t_1 - t_0\right) \\ 0 \\ 0 \end{bmatrix} & \text{if } t_3 \le t \end{cases}$$

Cosine Ramps

$t := 0, 0.001 .. t_3 + 0.1$





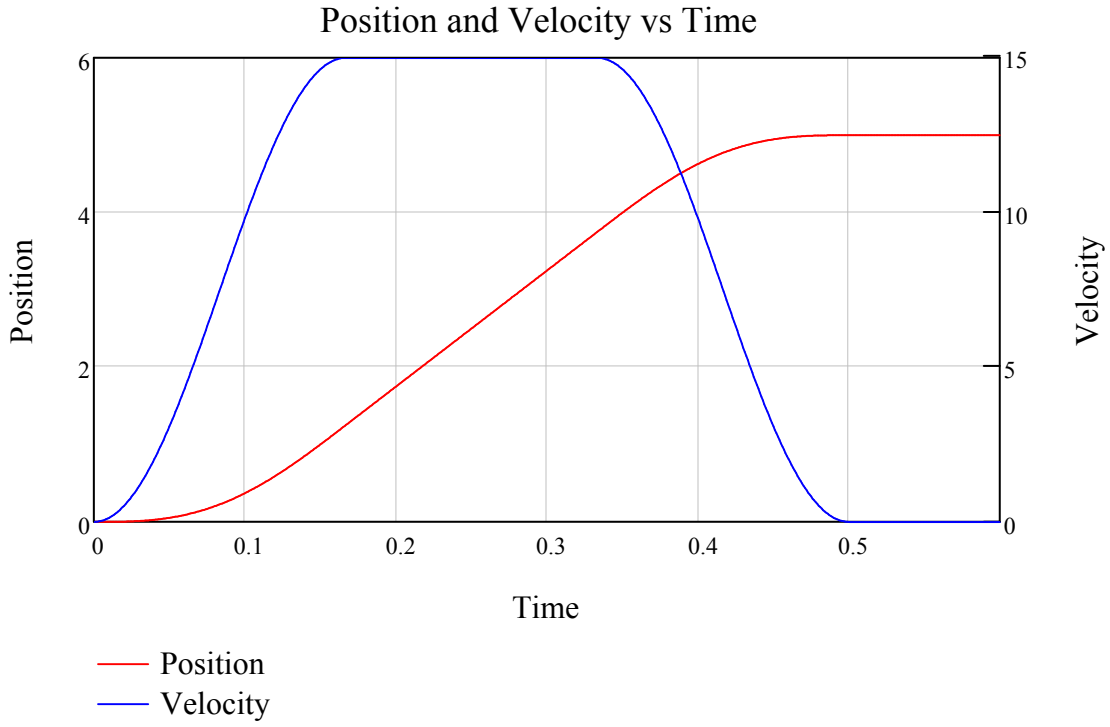
Cosine Ramps

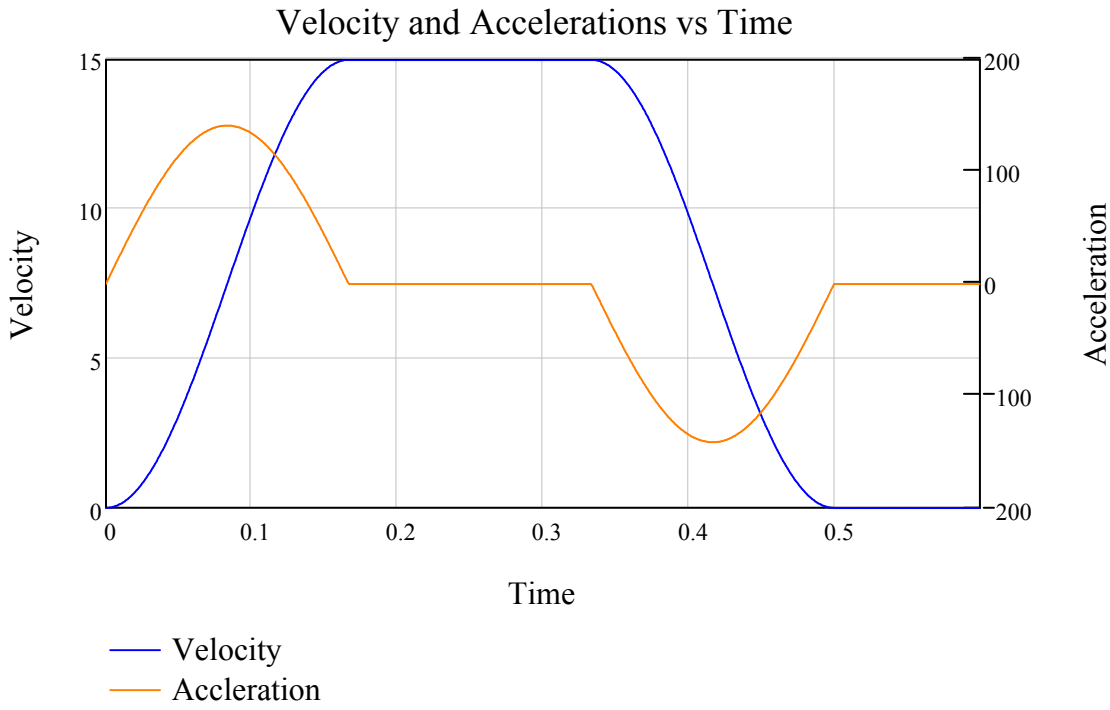
$\Delta t := 0.5 \qquad \Delta x := 5$

$$TGCos(t, \Delta t, \Delta x) := \begin{cases} \begin{bmatrix} \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(t - \frac{1}{3}\cdot\Delta t\cdot\frac{\sin\left(3\cdot\frac{\pi}{\Delta t}\cdot t\right)}{\pi}\right) \\ \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(1 - \cos\left(3\cdot\frac{\pi}{\Delta t}\cdot t\right)\right) \\ \frac{9}{4}\cdot\frac{\Delta x}{\Delta t^2}\cdot\sin\left(3\cdot\frac{\pi}{\Delta t}\cdot t\right)\cdot\pi \end{bmatrix} & \text{if } 0 \le t < \frac{\Delta t}{3} \\ \begin{bmatrix} \frac{3}{2}\cdot\frac{\Delta x}{\Delta t}\cdot\left(t - \frac{1}{3}\cdot\Delta t\right) + \frac{1}{4}\cdot\Delta x \\ \frac{3}{2}\cdot\frac{\Delta x}{\Delta t} \\ 0 \end{bmatrix} & \text{if } \frac{\Delta t}{3} \le t < 2\cdot\frac{\Delta t}{3} \\ \begin{bmatrix} \frac{1}{4}\cdot\frac{\Delta x}{\pi}\cdot\sin\left(3\cdot\pi\cdot\frac{t - \frac{2}{3}\cdot\Delta t}{\Delta t}\right) + \frac{3}{4}\cdot\left(t + \frac{1}{3}\cdot\Delta t\right)\cdot\frac{\Delta x}{\Delta t} \\ \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(1 + \cos\left(3\cdot\pi\cdot\frac{t - \frac{2}{3}\cdot\Delta t}{\Delta t}\right)\right) \\ \frac{-9}{4}\cdot\frac{\Delta x}{\Delta t^2}\cdot\sin\left(3\cdot\pi\cdot\frac{t - \frac{2}{3}\cdot\Delta t}{\Delta t}\right)\cdot\pi \end{bmatrix} & \text{if } 2\cdot\frac{\Delta t}{3} \le t < \Delta t \\ \begin{pmatrix} \Delta x \\ 0 \\ 0 \end{pmatrix} & \text{if } \Delta t \le t \end{cases}$$

Cosine Ramps

$t := 0, 0.001 .. \Delta t + 0.1$





Cosine Ramps

$$\mathrm{tgcos}(t,\Delta t,\Delta x) := \begin{cases} \begin{bmatrix} \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(t-\frac{1}{3}\cdot\Delta t\cdot\frac{\sin\left(3\cdot\frac{\pi}{\Delta t}\cdot t\right)}{\pi}\right) \\ \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(1-\cos\left(3\cdot\frac{\pi}{\Delta t}\cdot t\right)\right) \end{bmatrix} & \text{if } 0 \le t < \frac{\Delta t}{3} \\ \begin{bmatrix} \frac{3}{2}\cdot\frac{\Delta x}{\Delta t}\cdot\left(t-\frac{1}{3}\cdot\Delta t\right)+\frac{1}{4}\cdot\Delta x \\ \frac{3}{2}\cdot\frac{\Delta x}{\Delta t} \end{bmatrix} & \text{if } \frac{\Delta t}{3} \le t < 2\cdot\frac{\Delta t}{3} \\ \begin{bmatrix} \frac{1}{4}\cdot\frac{\Delta x}{\pi}\cdot\sin\left(3\cdot\pi\cdot\frac{t-\frac{2}{3}\cdot\Delta t}{\Delta t}\right)+\frac{3}{4}\cdot\left(t+\frac{1}{3}\cdot\Delta t\right)\cdot\frac{\Delta x}{\Delta t} \\ \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(1+\cos\left(3\cdot\pi\cdot\frac{t-\frac{2}{3}\cdot\Delta t}{\Delta t}\right)\right) \end{bmatrix} & \text{if } 2\cdot\frac{\Delta t}{3} \le t < \Delta t \\ \begin{pmatrix} \Delta x \\ 0 \end{pmatrix} & \text{if } \Delta t \le t \end{cases}$$

$$\left(\frac{1}{2}\cdot t_0\right)\cdot\frac{v}{\pi}\cdot\sin\left[t\cdot\frac{\pi}{\left(-t_1\right)+t_0}\right]+\left(\frac{1}{2}\cdot t-\frac{1}{2}\cdot t_0\right)\cdot v+x_0$$

$$\frac{v}{2}\cdot\sin\left(t_2\cdot\frac{\pi}{t_3-t_2}\right)+\left(\frac{1}{2}\cdot t-\frac{1}{2}\cdot t_2\right)\cdot v$$