Cosine Squared Ramps

velocity as a function of time

$$v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)^2$$

Position as a function of time

$$\int v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)^2 dt \text{ simplify } \rightarrow \frac{1}{4}\cdot\left(\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot T + 2\cdot\pi\cdot t\right)\cdot\frac{v_{max}}{\pi}$$

$$\int_0^{\frac{T}{4}} v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)^2 dt \rightarrow \frac{1}{8}\cdot T\cdot v_{max}$$

Acceleration as a function of time

$$\frac{d}{dt}\left(v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)^2\right) \rightarrow (-4)\cdot v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\frac{\pi}{T}$$

Jerk as a function of time

$$\frac{d^2}{dt^2}\left(v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)^2\right) \rightarrow 8\cdot v_{max}\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)^2\cdot\frac{\pi^2}{T^2} - 8\cdot v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)^2\cdot\frac{\pi^2}{T^2}$$

Distance covered

$$\Delta x = \frac{v_{max}\cdot T}{4} \text{ solve}, v_{max} \rightarrow 4\cdot\frac{\Delta x}{T}$$

Cosine Squared Ramps

velocity as a function of time

$$v_{max}\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)^2$$

Position as a function of time

$$\int v_{max}\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)^2 dt \text{ simplify } \rightarrow \frac{-1}{4}\cdot\left(\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot T - 2\cdot\pi\cdot t\right)\cdot\frac{v_{max}}{\pi}$$

$$\int_0^{\frac{T}{2}} v_{max}\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)^2 dt \rightarrow \frac{1}{4}\cdot T\cdot v_{max}$$

Acceleration as a function of time

$$\frac{d}{dt}\left(v_{max}\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)^2\right) \rightarrow 4\cdot v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\frac{\pi}{T}$$

maximum acceleration

$$4\cdot v_{max}\cdot\cos\left(\frac{\pi}{4}\right)\cdot\sin\left(\frac{\pi}{4}\right)\cdot\frac{\pi}{T} \left|\begin{array}{l}\text{simplify}\\ \text{explicit}\end{array}\right. \rightarrow 2\cdot v_{max}\cdot\frac{\pi}{T}$$

$$4\cdot v_{max}\cdot\cos\left(\frac{\pi}{4}\right)\cdot\sin\left(\frac{\pi}{4}\right)\cdot\frac{\pi}{T} \left|\begin{array}{l}\text{substitute}, v_{max} = 4\cdot\frac{\Delta x}{T}\\ \text{simplify}\\ \text{explicit}\end{array}\right. \rightarrow 8\cdot\frac{\Delta x}{T^2}\cdot\pi$$

Jerk as a function of time

$$\frac{d^2}{dt^2}\left(v_{max}\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)^2\right) \rightarrow 8\cdot v_{max}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)^2\cdot\frac{\pi^2}{T^2} - 8\cdot v_{max}\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)^2\cdot\frac{\pi^2}{T^2}$$

maximum jerk

$$j_{max} = 8\cdot\frac{\pi^2}{T^2}\cdot v_{max} \text{ substitute}, v_{max} = \frac{4\cdot\Delta x}{T} \rightarrow j_{max} = 32\cdot\frac{\pi^2}{T^3}\cdot\Delta x$$

Cosine Squared Ramps

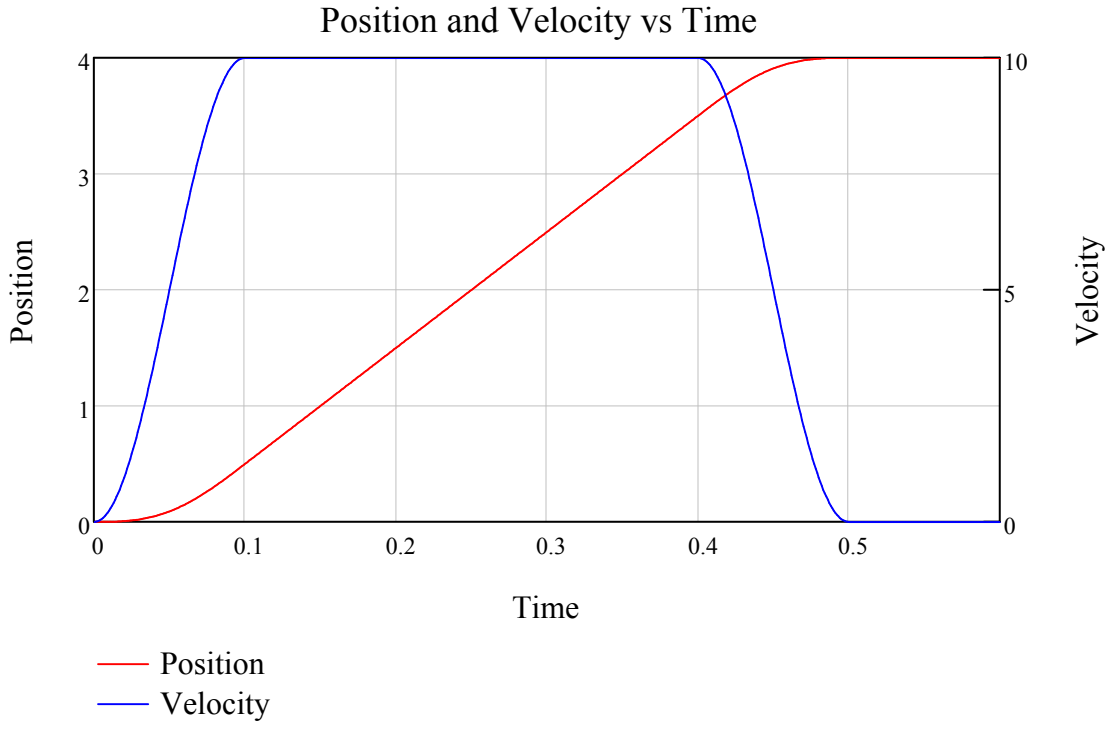
$x_0 := 0 \qquad v := 10$

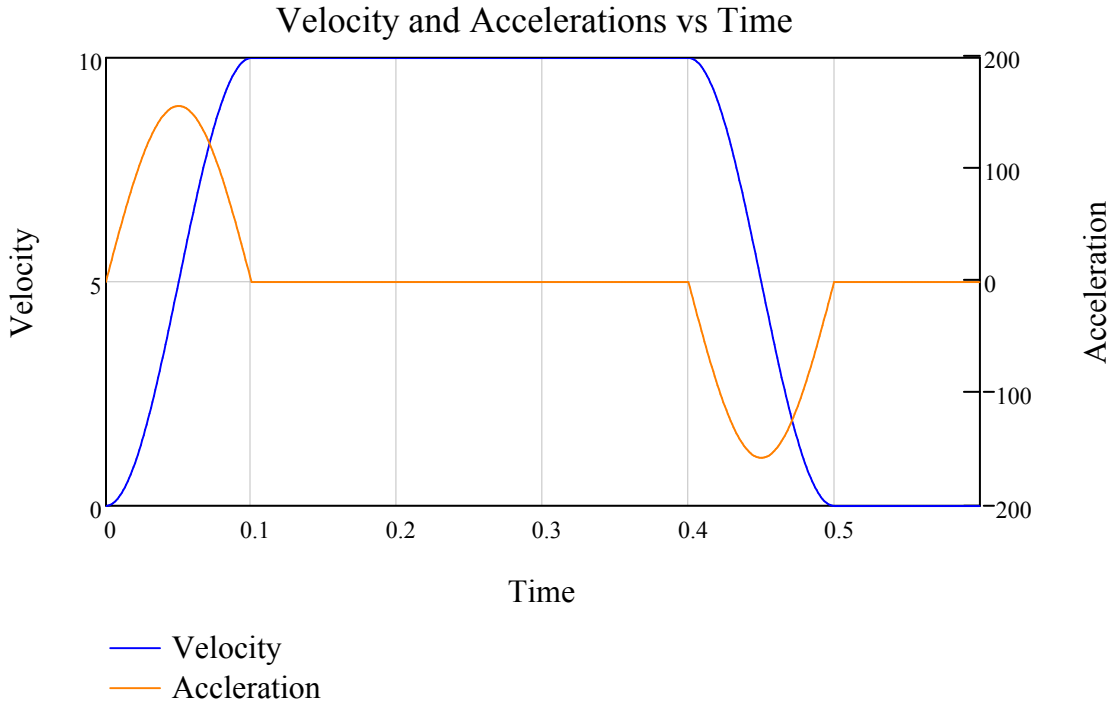
$t_0 := 0 \qquad t_1 := 0.1 \qquad t_2 := 0.4 \qquad t_3 := 0.5$

$$r(t) := \begin{cases} \begin{bmatrix} \frac{v}{2}\cdot\left[\left(t - t_0\right) - \frac{\left(t_1 - t_0\right)\cdot\sin\left(\pi\cdot\frac{t - t_0}{t_1 - t_0}\right)}{\pi}\right] + x_0 \\ \frac{v}{2}\cdot\left(1 - \cos\left(\pi\cdot\frac{t - t_0}{t_1 - t_0}\right)\right) \\ \frac{1}{2}\cdot v\cdot\sin\left(\pi\cdot\frac{t - t_0}{t_1 - t_0}\right)\cdot\frac{\pi}{t_1 - t_0} \end{bmatrix} & \text{if } t_0 \le t < t_1 \\ \begin{bmatrix} v\cdot\left(t - t_1\right) + x_0 + \frac{v}{2}\cdot t_1 \\ v \\ 0 \end{bmatrix} & \text{if } t_1 \le t < t_2 \\ \begin{bmatrix} \frac{v\cdot\left(t_3 - t_2\right)}{2\cdot\pi}\cdot\sin\left(\pi\cdot\frac{t - t_2}{t_3 - t_2}\right) + \left(t + t_2 - t_1\right)\cdot\frac{v}{2} + x_0 \\ \frac{v}{2}\cdot\left(1 + \cos\left(\pi\cdot\frac{t - t_2}{t_3 - t_2}\right)\right) \\ \frac{-1}{2}\cdot v\cdot\sin\left(\pi\cdot\frac{t - t_2}{t_3 - t_2}\right)\cdot\frac{\pi}{t_3 - t_2} \end{bmatrix} & \text{if } t_2 \le t < t_3 \\ \begin{bmatrix} \frac{v}{2}\cdot\left(t_3 + t_2 - t_1 - t_0\right) \\ 0 \\ 0 \end{bmatrix} & \text{if } t_3 \le t \end{cases}$$

Cosine Squared Ramps

$t := 0, 0.001 .. t_3 + 0.1$





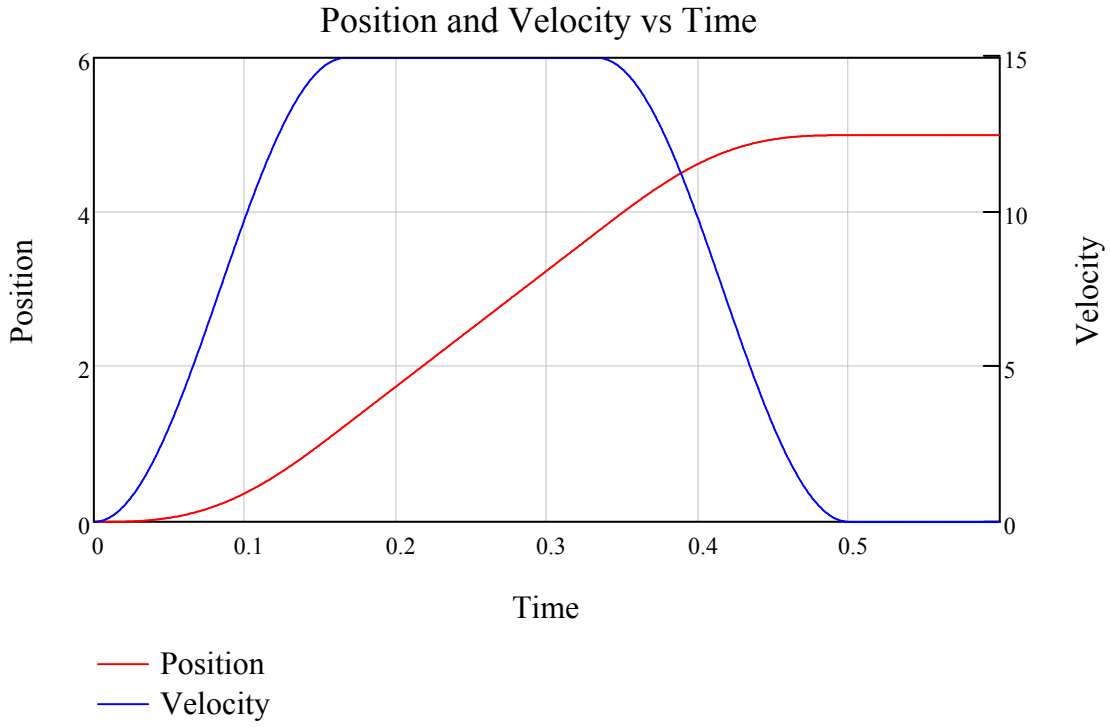
Cosine Squared Ramps

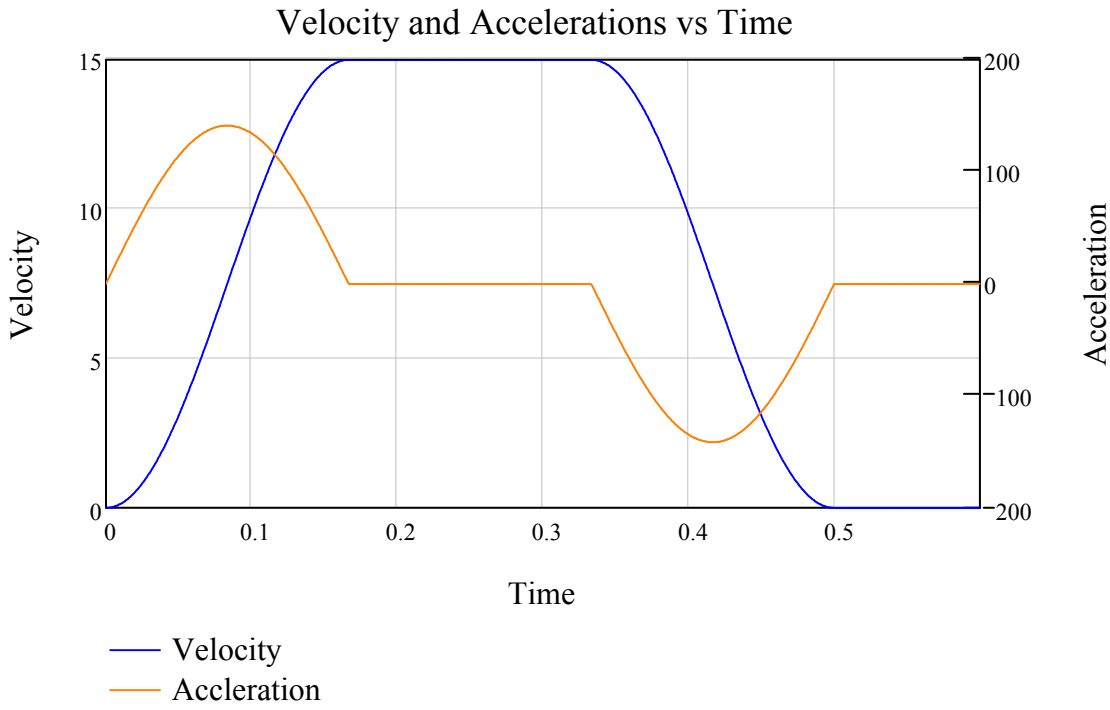
$\Delta t := 0.5 \qquad \Delta x := 5$

$$r(t, \Delta t, \Delta x) := \begin{cases} \begin{bmatrix} \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(t - \frac{1}{3}\cdot\Delta t\cdot\frac{\sin\left(3\cdot\frac{\pi}{\Delta t}\cdot t\right)}{\pi}\right) \\ \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(1 - \cos\left(3\cdot\frac{\pi}{\Delta t}\cdot t\right)\right) \\ \frac{9}{4}\cdot\frac{\Delta x}{\Delta t^2}\cdot\sin\left(3\cdot\frac{\pi}{\Delta t}\cdot t\right)\cdot\pi \end{bmatrix} & \text{if } 0 \le t < \frac{\Delta t}{3} \\ \begin{bmatrix} \frac{3}{2}\cdot\frac{\Delta x}{\Delta t}\cdot\left(t - \frac{1}{3}\cdot\Delta t\right) + \frac{1}{4}\cdot\Delta x \\ \frac{3}{2}\cdot\frac{\Delta x}{\Delta t} \\ 0 \end{bmatrix} & \text{if } \frac{\Delta t}{3} \le t < 2\cdot\frac{\Delta t}{3} \\ \begin{bmatrix} \frac{1}{4}\cdot\frac{\Delta x}{\pi}\cdot\sin\left(3\cdot\pi\cdot\frac{t - \frac{2}{3}\cdot\Delta t}{\Delta t}\right) + \frac{3}{4}\cdot\left(t + \frac{1}{3}\cdot\Delta t\right)\cdot\frac{\Delta x}{\Delta t} \\ \frac{3}{4}\cdot\frac{\Delta x}{\Delta t}\cdot\left(1 + \cos\left(3\cdot\pi\cdot\frac{t - \frac{2}{3}\cdot\Delta t}{\Delta t}\right)\right) \\ \frac{-9}{4}\cdot\frac{\Delta x}{\Delta t^2}\cdot\sin\left(3\cdot\pi\cdot\frac{t - \frac{2}{3}\cdot\Delta t}{\Delta t}\right)\cdot\pi \end{bmatrix} & \text{if } 2\cdot\frac{\Delta t}{3} \le t < \Delta t \\ \begin{pmatrix} \Delta x \\ 0 \\ 0 \end{pmatrix} & \text{if } \Delta t \le t \end{cases}$$

Cosine Squared Ramps

$t := 0, 0.001 .. \Delta t + 0.1$





Cosine Squared Ramps

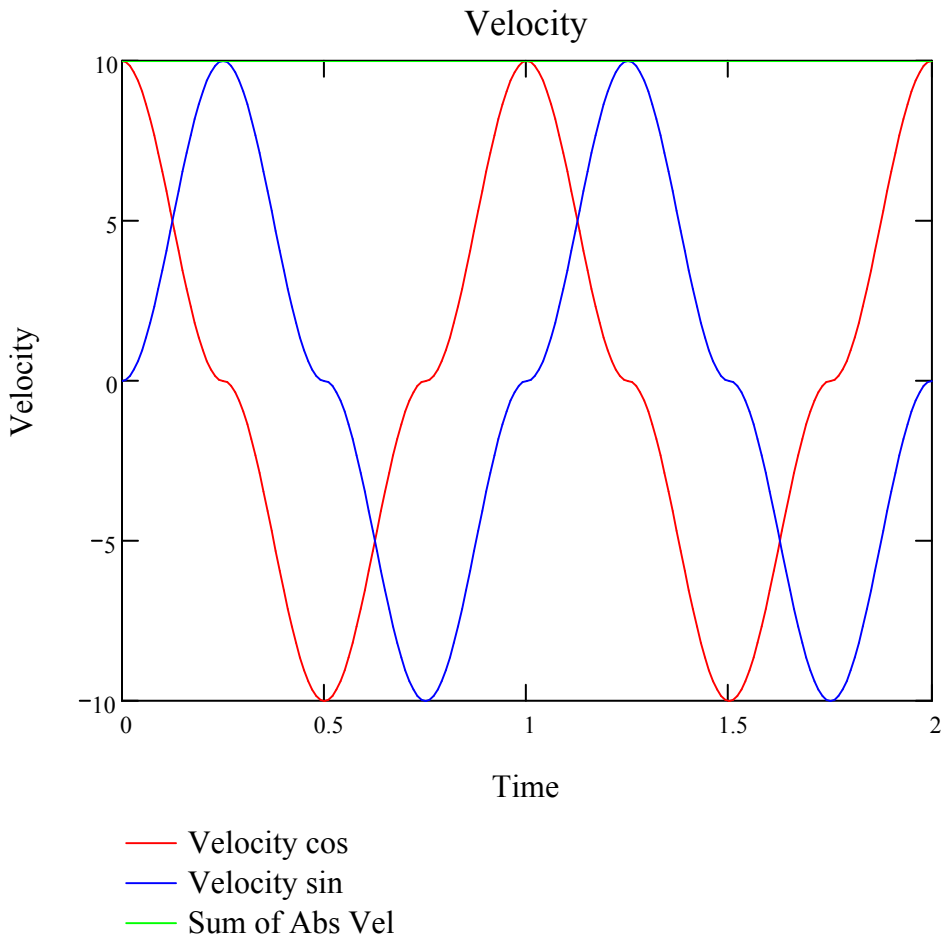
Velocity

$T := 1 \qquad v_{max} := 10$

$$v_c(t) := \begin{| t \leftarrow \mathrm{mod}(t, T) \\ v \leftarrow -v_{max} \cdot \cos\left(2 \cdot \pi \cdot \frac{t}{T}\right)^2 \text{ if } \frac{T}{4} \le t < \frac{3 \cdot T}{4} \\ v \leftarrow v_{max} \cdot \cos\left(2 \cdot \pi \cdot \frac{t}{T}\right)^2 \text{ otherwise}}$$

$$v_s(t) := \begin{| t \leftarrow \mathrm{mod}(t, T) \\ v \leftarrow v_{max} \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T}\right)^2 \text{ if } 0 \le t < \frac{T}{2} \\ v \leftarrow -v_{max} \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T}\right)^2 \text{ otherwise}}$$

$t := 0, 0.01 .. 2 \cdot T$

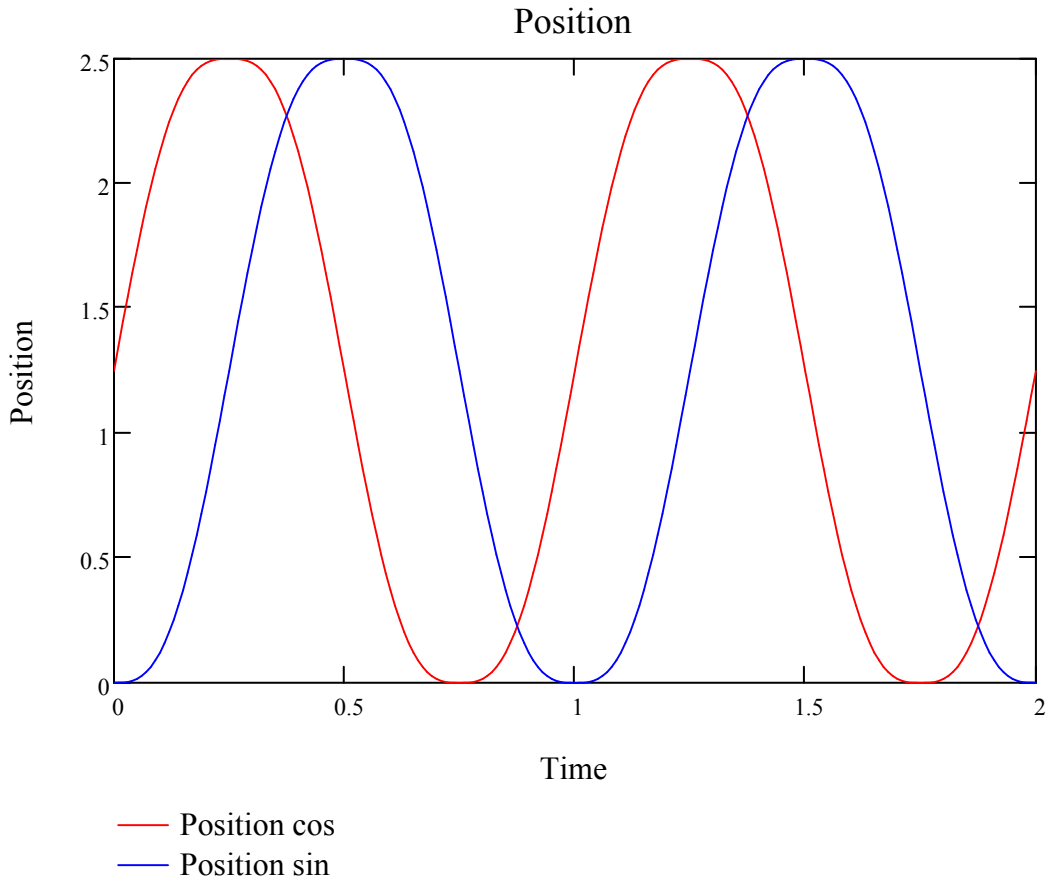


Cosine Squared Ramps

Position

$$x_c(t) := \left| \begin{array}{l} t \leftarrow \mathrm{mod}(t, T) \\ \left(\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot T + 2\cdot\pi\cdot t\right)\cdot\frac{v_{max}}{4\cdot\pi} + \frac{T}{8}\cdot v_{max} \quad \text{if } t < \frac{T}{4} \\ \left(\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot T + 2\cdot\pi\cdot t\right)\cdot\frac{-v_{max}}{4\cdot\pi} + \frac{3\cdot T}{8}\cdot v_{max} \quad \text{if } \frac{T}{4} \le t < \frac{3\cdot T}{4} \\ \left(\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot T + 2\cdot\pi\cdot t\right)\cdot\frac{v_{max}}{4\cdot\pi} - \frac{3T}{8}\cdot v_{max} \quad \text{if } \frac{3\cdot T}{4} \le t \end{array} \right.$$

$$x_s(t) := \left| \begin{array}{l} t \leftarrow \mathrm{mod}(t, T) \\ \left(2\cdot\pi\cdot t - \cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot T\right)\cdot\frac{v_{max}}{4\cdot\pi} \quad \text{if } t < \frac{T}{2} \\ \left(2\cdot\pi\cdot t - \cos\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)\cdot T\right)\cdot\frac{-v_{max}}{4\cdot\pi} + \frac{T}{2}\cdot v_{max} \quad \text{if } \frac{T}{2} \le t \end{array} \right.$$

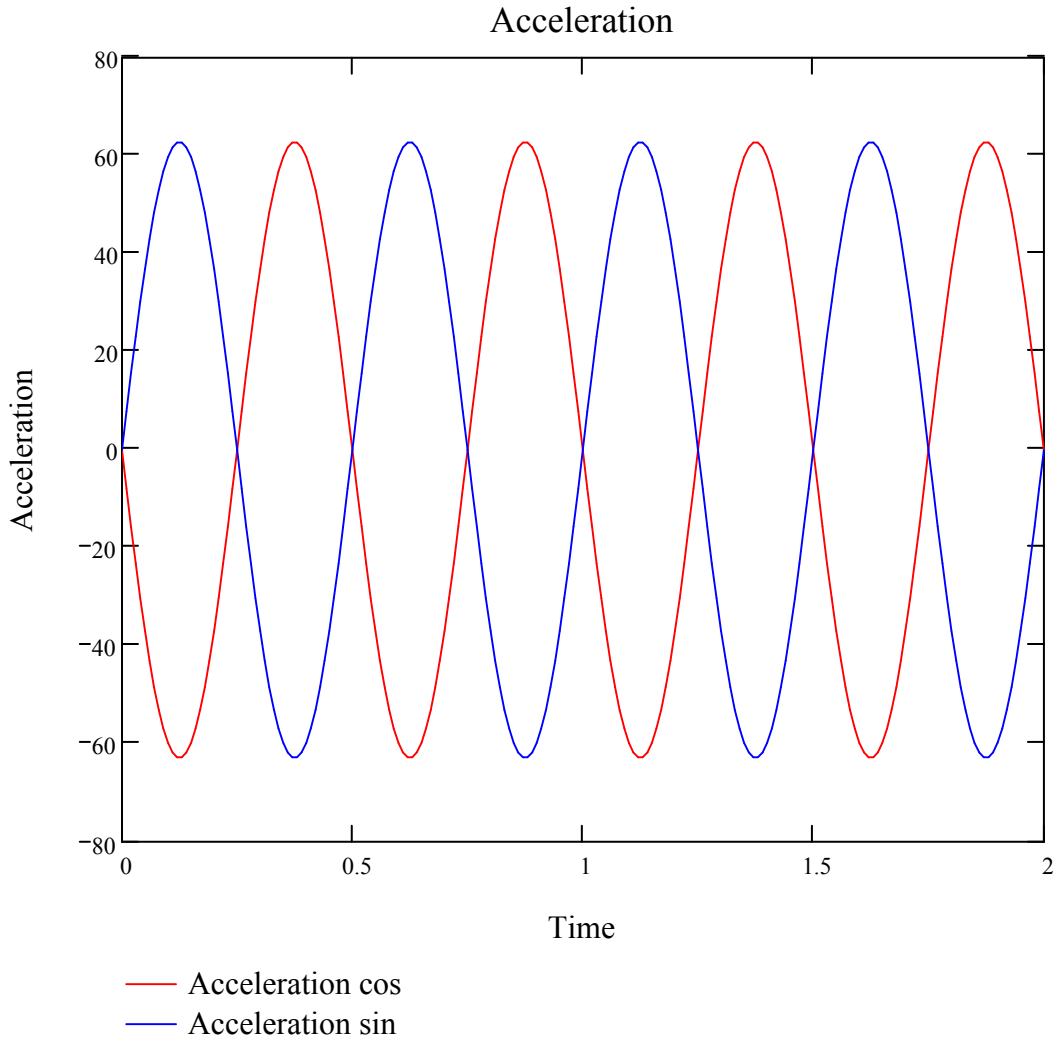


Cosine Squared Ramps

Acceleration

$$a_c(t) := \left| \begin{array}{l} t \leftarrow \mathrm{mod}(t, T) \\ -4 \cdot v_{max} \cdot \cos\left(2 \cdot \pi \cdot \frac{t}{T}\right) \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T}\right) \cdot \frac{\pi}{T} \end{array} \right.$$

$$a_s(t) := \left| \begin{array}{l} t \leftarrow \mathrm{mod}(t, T) \\ 4 \cdot v_{max} \cdot \cos\left(2 \cdot \pi \cdot \frac{t}{T}\right) \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T}\right) \cdot \frac{\pi}{T} \end{array} \right.$$



Cosine Squared Ramps

Jerk

$$j_c(t) := \left| \begin{array}{l} t \leftarrow \mathrm{mod}(t, T) \\ 8 \cdot v_{max} \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T}\right)^2 \cdot \frac{\pi^2}{T^2} - 8 \cdot v_{max} \cdot \cos\left(2 \cdot \pi \cdot \frac{t}{T}\right)^2 \cdot \frac{\pi^2}{T^2} \end{array} \right.$$

$$j_s(t) := \left| \begin{array}{l} t \leftarrow \mathrm{mod}(t, T) \\ 8 \cdot v_{max} \cdot \cos\left(2 \cdot \pi \cdot \frac{t}{T}\right)^2 \cdot \frac{\pi^2}{T^2} - 8 \cdot v_{max} \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T}\right)^2 \cdot \frac{\pi^2}{T^2} \end{array} \right.$$