Lead Lag Control of a Type 1 Single Pole System

Lead or Lag Control of a Type 1 System with One Real Pole

$$G_a(s) = \frac{K_a \cdot \alpha}{s \cdot (s + \alpha)} = \frac{K}{s \cdot (\tau \cdot s + 1)} \qquad 1$$

$G_a(s)$, a type 1 system (integrating) with a bandwidth of α or time constant of $\tau=1/\alpha$

$$G_c(s) = \frac{K_c \cdot (s + sZero)}{s + sPole} \qquad 2$$

$G_c(s)$, a lead or lag filter depending on the value of the zero and pole.

$$CLTF(s) = \frac{\dfrac{K_c \cdot (s + sZero)}{s + sPole} \cdot \dfrac{K_a \cdot \alpha}{s \cdot (s + \alpha)}}{1 + \dfrac{K_c \cdot (s + sZero)}{s + sPole} \cdot \dfrac{K_a \cdot \alpha}{s \cdot (s + \alpha)}} \qquad 3$$

Closed loop transfer function

$$CLTF(s) = \frac{G_a(s) \cdot G_c(s)}{1 + G_a(s) \cdot G_c(s)} \qquad 4$$

Closed loop transfer function with the characteristic equation in powers if s.
The CLTF will have 1 zero and 3 poles.

$$CLTF(s) = \frac{K_c \cdot (s + sZero) \cdot K_a \cdot \alpha}{s^3 + (sPole + \alpha) \cdot s^2 + (K_c \cdot K_a \cdot \alpha + sPole \cdot \alpha) \cdot s + K_c \cdot K_a \cdot \alpha \cdot sZero} \qquad 5$$

$$(s + \lambda)^3 \text{ collect}, s \;\rightarrow\; s^3 + 3 \cdot \lambda \cdot s^2 + 3 \cdot \lambda^2 \cdot s + \lambda^3 \qquad 6$$

Desired characteristic equation.
Place 3 real poles at $-\lambda$.

$$s^3 + (sPole + \alpha) \cdot s^2 + (K_c \cdot K_a \cdot \alpha + sPole \cdot \alpha) \cdot s + K_c \cdot K_a \cdot \alpha \cdot sZero \text{ coeffs}, s \;\rightarrow\; \begin{pmatrix} K_c \cdot K_a \cdot \alpha \cdot sZero \\ K_c \cdot K_a \cdot \alpha + sPole \cdot \alpha \\ sPole + \alpha \\ 1 \end{pmatrix} \qquad 7$$

$$(s + \lambda)^3 \text{ coeffs}, s \;\rightarrow\; \begin{pmatrix} \lambda^3 \\ 3 \cdot \lambda^2 \\ 3 \cdot \lambda \\ 1 \end{pmatrix} \qquad 8$$

Given

$\lambda^3 = K_c \cdot K_a \cdot \alpha \cdot sZero$

$3 \cdot \lambda^2 = K_c \cdot K_a \cdot \alpha + sPole \cdot \alpha$

$3 \cdot \lambda = sPole + \alpha$

Match the coefficients for each power of s and solve for the controller parameters.

$$\text{Find}(K_c, sPole, sZero) \rightarrow \begin{pmatrix} \dfrac{3\cdot\lambda^2 - 3\cdot\alpha\cdot\lambda + \alpha^2}{K_a\cdot\alpha} \\ 3\cdot\lambda - \alpha \\ \dfrac{\lambda^3}{3\cdot\lambda^2 - 3\cdot\alpha\cdot\lambda + \alpha^2} \end{pmatrix}$$ 9

Find the difference between the system characteristic equation and the desired characteristic equation and the solve for the gains. This is my "short hand" way of solving to the controller parameters. This is a more general solution because there is one real closed loop poles and a complex,pair of poles.

$$DiffCE := s^3 + (sPole + \alpha)\cdot s^2 + (K_c\cdot K_a\cdot\alpha + sPole\cdot\alpha)\cdot s + K_c\cdot K_a\cdot\alpha\cdot sZero - (s + \lambda)\cdot\left[(s + \mu)^2 + \nu^2\right]$$ 10

$$(K_c \ \ sPole \ \ sZero) := DiffCE \left| \begin{matrix} \text{coeffs}, s \\ \text{solve}, \begin{pmatrix} K_c \\ sPole \\ sZero \end{pmatrix} \end{matrix} \right. \rightarrow \begin{bmatrix} \dfrac{-\left[(-\mu^2) - \alpha^2 + 2\cdot\alpha\cdot\mu + \alpha\cdot\lambda - \nu^2 - 2\cdot\lambda\cdot\mu\right]}{K_a\cdot\alpha} \\ (-\alpha) + 2\cdot\mu + \lambda \\ (-\lambda)\cdot\dfrac{\mu^2 + \nu^2}{(-\mu^2) - \alpha^2 + 2\cdot\alpha\cdot\mu + \alpha\cdot\lambda - \nu^2 - 2\cdot\lambda\cdot\mu} \end{bmatrix}$$ 11

$$DiffCE := s^3 + (sPole + \alpha)\cdot s^2 + (K_c\cdot K_a\cdot\alpha + sPole\cdot\alpha)\cdot s + K_c\cdot K_a\cdot\alpha\cdot sZero - (s + \lambda)^2\cdot(s + \alpha)$$ 12

System parameters

$K_a := 2$

Open loop gain. (mm/s)/% control output. Max speed is 200 mm/s

$\alpha := 2$ $\alpha = 2$

$\tau := \frac{1}{\alpha}$ $\tau = 0.5$

Open loop corner frequency or bandwidth. Convert frequency to radians per second. This motor and load has a very low bandwidth to simulate moving a large load.

$\lambda := 10 \cdot \alpha$ $\lambda = 20$

Place 3 closed loop poles at -λ. λ is in radians per second.

Calculate the Controller Gain, Pole and Zero in terms of the system parameters and the desired response, λ.

$K_c := \frac{3 \cdot \lambda^2 - 3 \cdot \alpha \cdot \lambda + \alpha^2}{K_a \cdot \alpha}$ $K_c = 271$ 13

$sPole := 3 \cdot \lambda - \alpha$ $sPole = 58$ 14

$sZero := \frac{\lambda^3}{3 \cdot \lambda^2 - 3 \cdot \alpha \cdot \lambda + \alpha^2}$ $sZero = 7.38$ 15

Feed Forwards

$Kv := \frac{1}{K_a}$ $Kv = 0.5$ Velocity feed forward %/(in/s) 16

$Ka := \frac{1}{K_a \cdot \alpha}$ $Ka = 0.25$ Acceleration feed forward %/(in/s^2) 17

Simulation

Convert controller's pole and zero in the s domain to the z domain.

$T := 0.001$

Controller update period, seconds

$zPole := \exp(-sPole \cdot T)$ $\quad zPole = 0.944$ $\quad zZero := \exp(-sZero \cdot T)$ $\quad zZero = 0.993$

Calculate the discrete controller gain

$$K_d := \frac{K_c \cdot sZero \cdot (1 - zPole)}{sPole \cdot (1 - zZero)} \quad K_d = 264.264$$

The gain for the difference equation must be adjusted so the steady state gain remains the same.

Simulation

Define the continuous time state space arrays

$$A_c := \begin{pmatrix} 0 & 1 \\ 0 & -\alpha \end{pmatrix} \qquad A_c = \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix}$$

$$B_c := \begin{pmatrix} 0 \\ K_a \cdot \alpha \end{pmatrix} \qquad B_c = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

Some often used definitions

$I := \text{identity}(2)$ 2x2 identity matrix

Calculate arrays for use in discrete time.

$$A := I + \sum_{n=1}^{7} \frac{{A_c}^n \cdot T^n}{n!} \qquad B := \left(I + \sum_{n=1}^{7} \frac{{A_c}^n \cdot T^n}{(n+1)!} \right) \cdot B_c \cdot T$$

$$A = \begin{pmatrix} 1 & 0.000999 \\ 0 & 0.998002 \end{pmatrix} \qquad B = \begin{pmatrix} 1.998667 \times 10^{-6} \\ 0.003996 \end{pmatrix}$$

Lead/Lag Control

$x_0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} \text{Position} \\ \text{Velocity} \end{pmatrix}$

Initial state.

$u_0 := 0$

Initial control output

$err_0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Initial integrator contribution to the control output

$$LL(r,x,u,err) := \left| \begin{array}{l} x_1 \leftarrow A \cdot x + B \cdot u \\ err \leftarrow \begin{pmatrix} r - x_0 \\ err_0 \end{pmatrix} \\ u_1 \leftarrow \max\left[\min\left[zPole \cdot u + K_d \cdot \left(err_0 - zZero \cdot err_1\right), 100\right], -100\right] \\ \begin{pmatrix} x_1 \\ u_1 \\ err \end{pmatrix} \end{array} \right.$$

$N := \frac{10}{T}$ $n := 0..N$ $N = 1 \times 10^4$

Simulate 10 seconds

$$r_n := \left| \begin{array}{l} 0 \text{ if } n < 0.01 \cdot N \\ 10 \text{ if } 0.01 \cdot N \le n \wedge n < 0.3 \cdot N \\ -10 \text{ if } 0.3 \cdot N \le n \wedge n < 0.7 \cdot N \\ 0 \text{ otherwise} \end{array} \right.$$

Calculate the target positions and velocities. In this case the target velocities are 0 since that target positions is making step jumps.

$$\begin{pmatrix} x_{n+1} \\ u_{n+1} \\ err_{n+1} \end{pmatrix} := LL\left(r_n, x_n, u_n, err_n\right)$$

Compute the system response and control output

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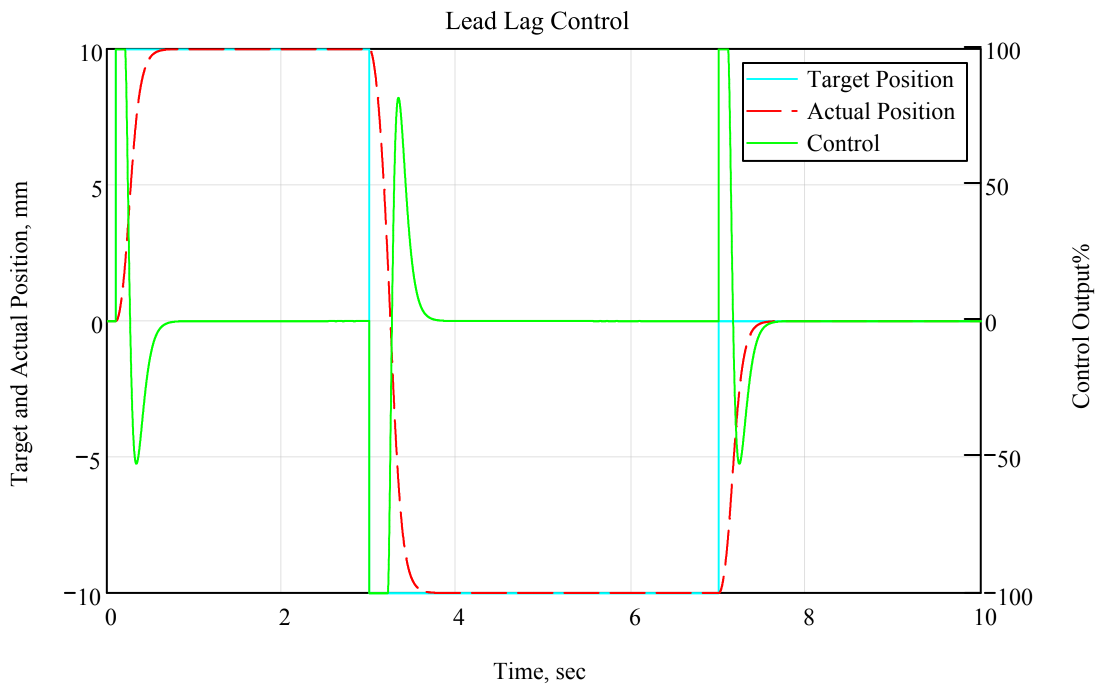

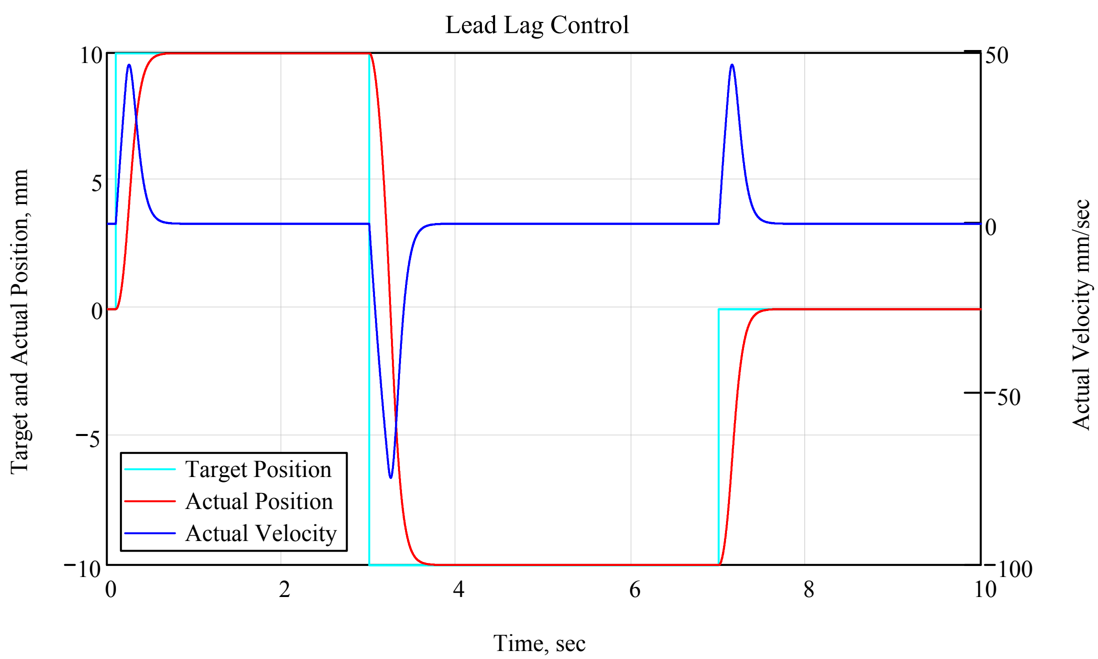

The controller handles output saturation well and doesn't over shoot because there is no integrator to wind up.

Cosine Ramps

Reference:C:\Documents and Settings\Peter\My Documents\mcd\tg\Cosine Ramps.xmcdz

$\Delta x := 10$ Move distance

$\Delta t := 1$ Move time

$$LL(r,x,u,err) := \left| \begin{array}{l} x_1 \leftarrow A\cdot x + B\cdot\left(u + r_1\cdot Kv + r_2\cdot Ka\right) \\ err \leftarrow \begin{pmatrix} r_0 - x_0 \\ err_0 \end{pmatrix} \\ u_1 \leftarrow \max\left[\min\left[zPole\cdot u + K_d\cdot\left(err_0 - zZero\cdot err_1\right), 100\right], -100\right] \\ \begin{pmatrix} x_1 \\ u_1 \\ err \end{pmatrix} \end{array} \right.$$

Added feed forwards.

$$n := 0 .. \frac{\Delta t + 0.1}{T}$$

$$r_n := TGCos\left(n\cdot T, \Delta t, \Delta x\right)$$

$$\begin{pmatrix} x_{n+1} \\ u_{n+1} \\ err_{n+1} \end{pmatrix} := LL\left(r_n, x_n, u_n, err_n\right)$$

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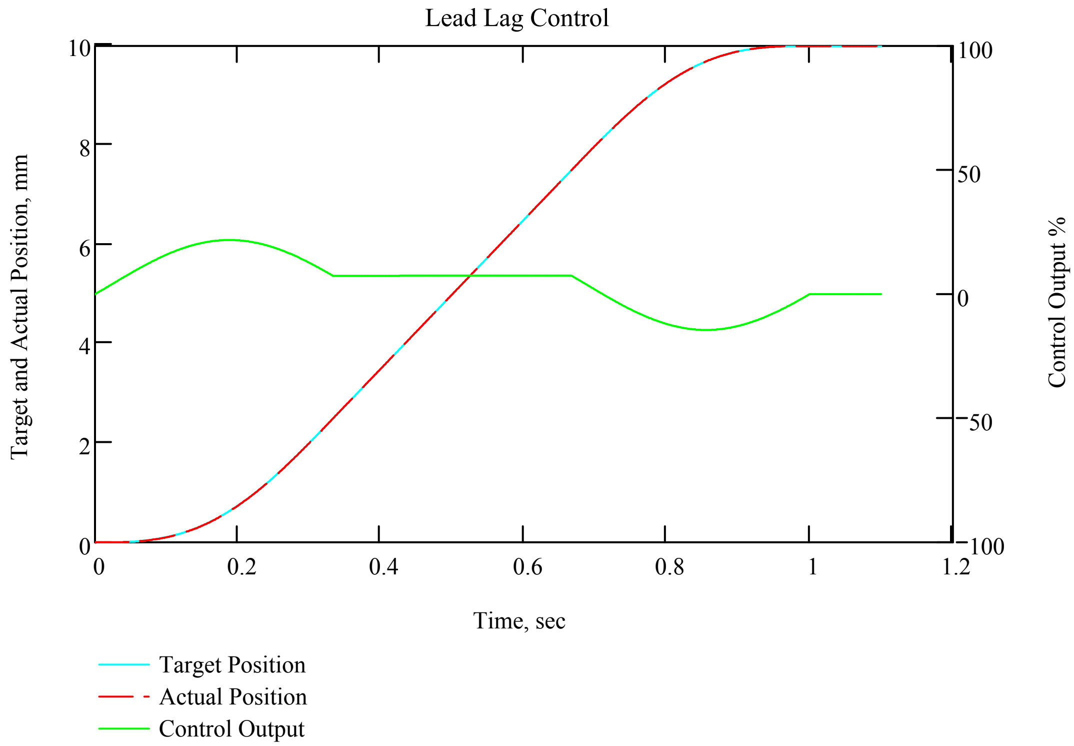
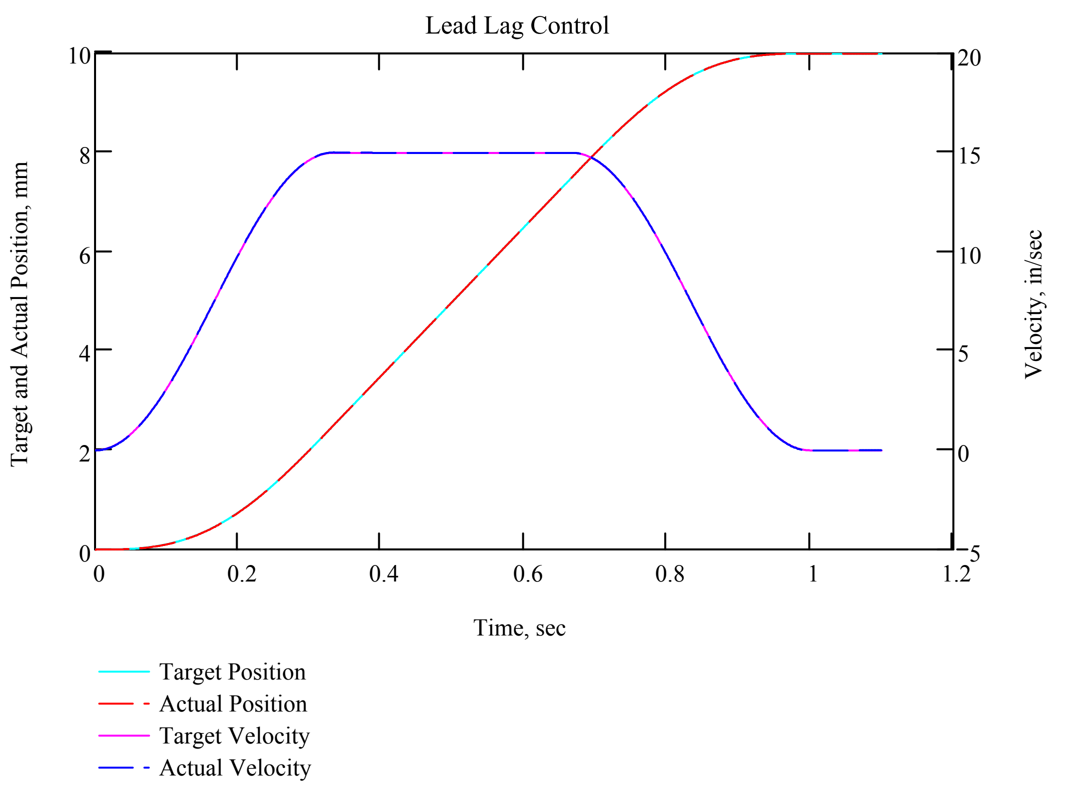




Closed Loop Lead or Lag Filter Bode Plot Calculations

$$CLTF(s) := \frac{K_c \cdot (s + sZero) \cdot K_a \cdot \alpha}{s^3 + (sPole + \alpha) \cdot s^2 + (K_c \cdot K_a \cdot \alpha + sPole \cdot \alpha) \cdot s + K_c \cdot K_a \cdot \alpha \cdot sZero}$$

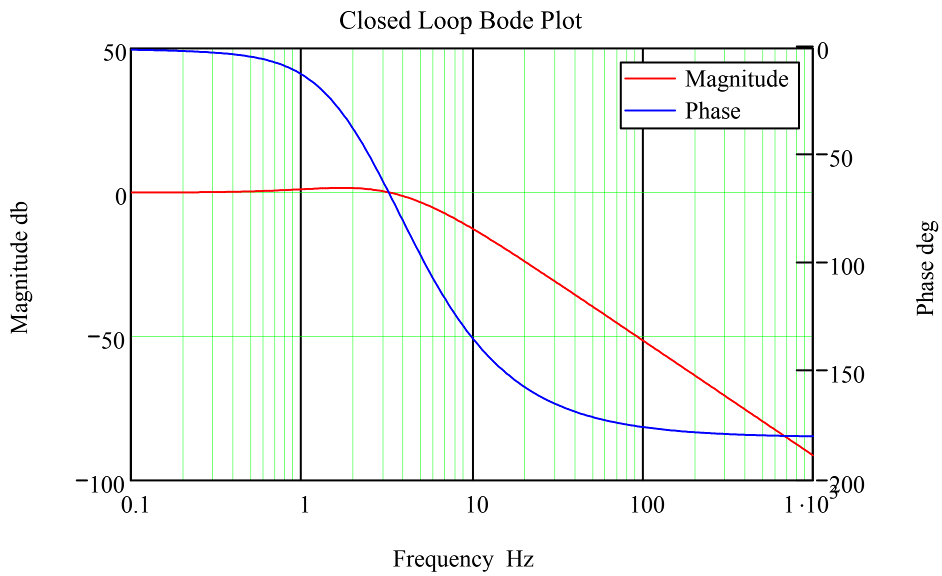
$n := 0 .. 256$

Iterate the frequency from 0.1 HZ to 1000 HZ using 64 steps per decade for a total of 256 iterations.

$$hz_n := 10^{\frac{n}{64} - 1}$$

$$\begin{pmatrix} M_n \\ \phi_n \end{pmatrix} := \left| \begin{array}{l} r \leftarrow CLTF\left[j \cdot (2 \cdot \pi) \cdot hz_n\right] \\ a \leftarrow \dfrac{\arg(r)}{\deg} \\ a \leftarrow a - 360 \quad \text{if } n > 0 \wedge \left|a - \phi_{n-1}\right| > 100 \\ \begin{pmatrix} 20 \cdot \log(|r|) \\ a \end{pmatrix} \end{array} \right.$$

Calculate magnitude and phase for T(s)



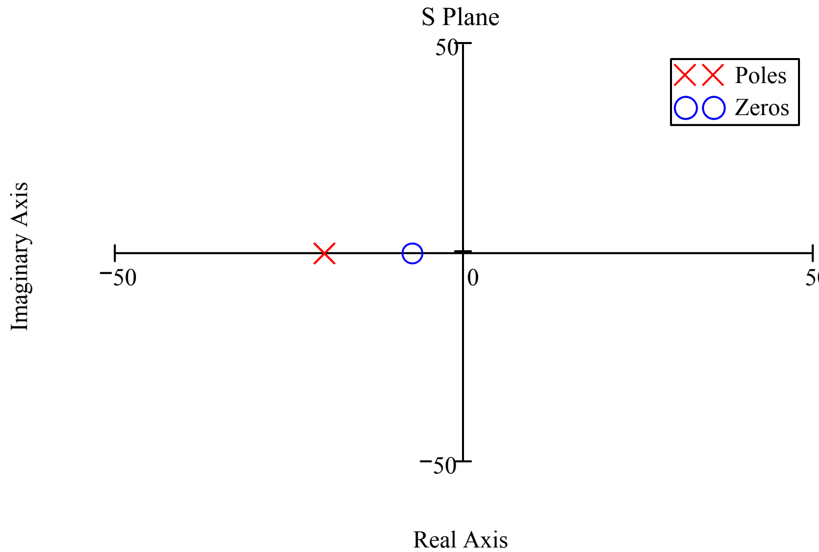
$hz := 100 \qquad bw := root\left[\left|CLTF\left[j \cdot (2 \cdot \pi) \cdot hz\right]\right| - \dfrac{1}{\sqrt{2}}, hz, 0.1, 1000\right] \qquad bw = 4.818 \quad Hz$

$bw \cdot 2 \cdot \pi = 30.275 \quad \dfrac{rad}{s}$

Graph Poles and Zeros

$$poles := polyroots\left(\begin{pmatrix} K_c \cdot K_a \cdot \alpha \cdot sZero \\ K_c \cdot K_a \cdot \alpha + sPole \cdot \alpha \\ sPole + \alpha \\ 1 \end{pmatrix}\right) \qquad poles = \begin{pmatrix} -20 \\ -20 \\ -20 \end{pmatrix} \qquad \lambda = 20$$

$$zeros := polyroots\left(\begin{pmatrix} sZero \\ 1 \end{pmatrix}\right) \qquad zeros = -7.38$$



Lead Lag Control of a Type 1 Single Pole System

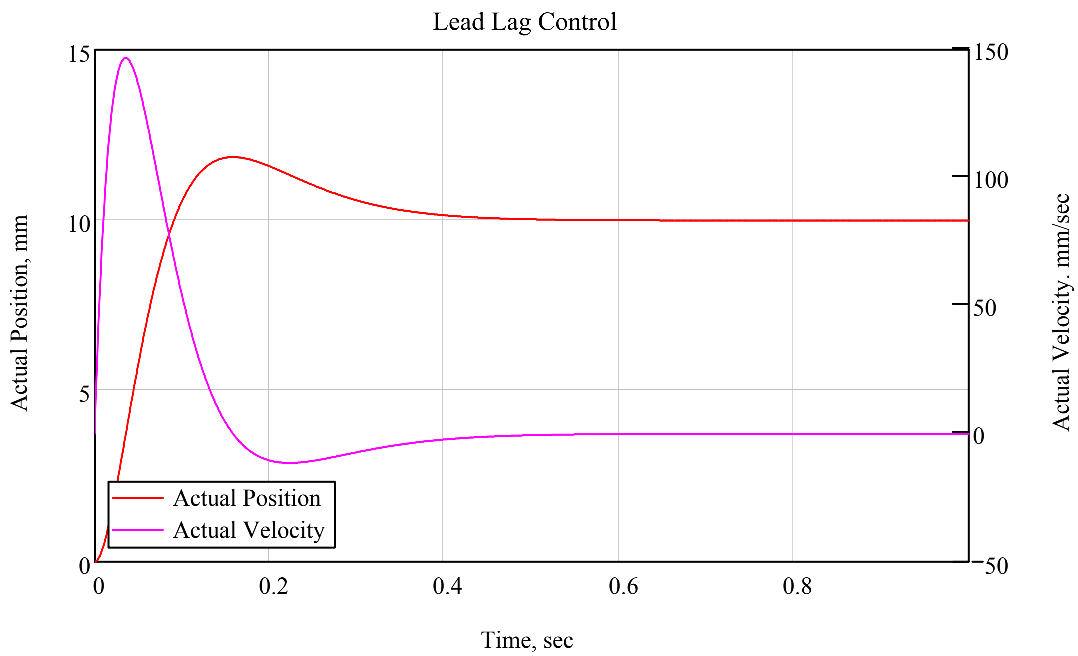
Do a step jump in the time domain Using Inverse Laplance Transform

$r := 10$ reference

$$\frac{r}{s}\cdot\frac{K_c\cdot(s + sZero)\cdot K_a\cdot\alpha}{(s+\lambda)^3} \left| \begin{array}{l} \text{invlaplace}, s \\ \text{explicit} \\ \text{collect}, r, K_c, K_a, e^{-\lambda\cdot t}, sZero \end{array} \right. \rightarrow \left[\left[\alpha\cdot\left(\frac{-1}{\lambda^3} - \frac{1}{\lambda^2}\cdot t - \frac{1}{2\cdot\lambda}\cdot t^2\right)\cdot sZero + \frac{1}{2}\cdot\alpha\cdot t^2\right]\cdot e^{(\cdot}\right.$$

$$pos(t) := \left[\left[\alpha\cdot\left(\frac{-1}{\lambda^3} - \frac{1}{\lambda^2}\cdot t - \frac{1}{2\cdot\lambda}\cdot t^2\right)\cdot sZero + \frac{1}{2}\cdot\alpha\cdot t^2\right]\cdot e^{(-\lambda)\cdot t} + \frac{\alpha}{\lambda^3}\cdot sZero\right]\cdot K_a\cdot K_c\cdot r$$

$t := 0, 0.001 .. 1$



Notice the overshoot. This happens because of the zero and there is no control output limitation.

Given

$$t_p = \frac{\pi}{\omega_n \cdot \sqrt{1 - \zeta^2}}$$

$$M_{\%p} = e^{-\frac{\zeta \cdot \pi}{\sqrt{1-\zeta^2}}}$$

$$\text{Find}(\zeta, \omega_n) \rightarrow \begin{bmatrix} \left(-\ln\left(M_{\%p}\right)\right) \cdot \dfrac{\left(\dfrac{\pi^2}{\ln\left(M_{\%p}\right)^2 + \pi^2}\right)^{\frac{1}{2}}}{\pi} \\ \dfrac{\pi}{t_p \cdot \left(\dfrac{\pi^2}{\ln\left(M_{\%p}\right)^2 + \pi^2}\right)^{\frac{1}{2}}} \end{bmatrix}$$

https://youtu.be/aalblaueedk?si=aCqG1u25fmbn42PS

$$\frac{4}{s\cdot(s+2)} \qquad \frac{2\cdot 2}{s\cdot(s+2)} \qquad 2\cdot\frac{2}{s+2}$$

$$- \lambda)\cdot t + \frac{\alpha}{\lambda^3}\cdot sZero\Bigg]\cdot K_a\cdot K_c\cdot r$$